Teichmüller space interpretation of quantum mechanics

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ABSTRACT. It is proposed that the non-local quantum mechanical correlations are explained by the analytic continuation below the Planck length into a complex Teichmüller space. In one space dimension, sufficient to explain the Einstein-Podolski-Rosen (EPR) paradox, the Teichmüller space is reduced to a space of complex Riemann surfaces, and an experiment is proposed to demonstrate the possible existence of such a space below the Planck length..

1 Introduction

To this day there is no satisfactory explanation for quantum mechanics. In *The Meaning of Relativity* Einstein wrote [1]:

One can give good reasons why reality cannot at all be represented by a continuous field. From the quantum phenomena it appears to follow with certainty that a finite system of finite energy can be completely described by a finite set of numbers (quantum numbers). This does not seem to be in accordance with a continuum theory and must lead to an attempt to find a purely algebraic theory for the representation of reality. But nobody knows how to find the basis for such a theory.

While the Riemannian space of general relativity describes reality in the large, the conjecture that a Teichmüller space can describe reality in the small is interesting for the following reason: Quantum mechanics has in a very fundamental way to do with the problem of measurement. But any measurement must be expressed in terms of rational numbers, with the prime numbers the basic building blocks of all rational numbers. In this regard it is remarkable that a deep connection between the Teichmüller theory [2] and number theory has most recently been discovered by S. Mochizuki in his groundbreaking work "Inter-Universal Teichmüller Theory," which is an arithmetic version of the Teichmüller theory for number fields with an elliptic curve [3].

The often-expressed view that at the Planck length space-time is a multiply connected "foam" [4], is contradicted by Redmount and Suen [5], who showed that such a configuration is unstable. Alternatively, it was originally proposed by A. Sakharov [6], and in much greater detail worked out by the author [7, 8, 9], that space might be filled with an equal number of positive and negative Planck mass particles (maximons), with one maximon per Planck length volume. And as it was shown by Redington [10], this configuration (Planck mass plasma) has stable space-time-rippling solutions of Einstein's gravitational field equation. The assumption of an equal amount of positive and negative masses also satisfies the null energy condition, the only one consistent with the conservation of mass-energy for all times.

One positive mass Planck mass particle is in essence a small black hole with a space-time singularity at its center. But in the presence of negative masses this singularity can be avoided by Einstein-Rosen bridges (wormholes). And with the Planck length $l=\sqrt{\hbar G/c^3}\sim 10^{-33}$ cm, there are $l^{-3}\sim 10^{99}$ cm^{-3} wormholes connecting the world above the Planck length to the world below this length, which opens the possibility to contemplate the analytic continuation of the wave function below the Planck length. For this reason it is conjectured that quantum mechanics may emerge from a Teichmüller space below the Planck length.

2 Quantum mechanics in one dimension and the EPR paradox

For quantum mechanics in one dimension and for an arbitrary number of particles, the Teichmüller space is reduced to the space of complex Riemann surfaces. While a one-dimensional space does not have the interference pattern of the double slit experiment, it already leads to the Einstein-Podolski-Rosen (EPR) paradox, which is most difficult to reconcile with any classical concept.

For one particle and in one dimension, quantum mechanics is described by the Schrödinger equation in one dimension

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi, \qquad (1)$$

$$\nabla^2=\frac{d^2}{dx^2}$$

where $\psi=\psi(x,t)$ is a complex function. For two particles it must be described in two dimensions, x_1, x_2, by the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi,\quad \nabla^2=\frac{\partial^2}{\partial x_1{}^2}+\frac{\partial^2}{\partial x_2{}^2} \qquad (2)$$

The two dimensions x_1, x_2 are not "real," but rather are an abstract two-dimensional configuration space, which unlike "ordinary" space cannot be visualized. The curved space-time of general relativity can at least be visualized by a curved surface. To visualize the complex Teichmüller space is by comparison much more difficult.

Even though the Schrödinger wave function is complex, its independent variables of space and time are real. As a first step to avoid this "onesidedness" we analytically continue the real space variable x into the complex plane putting

$$\left.\begin{aligned} x &\to x+iy=z \\ \psi(x) &\to u+iv=w(z) \end{aligned}\right\} \qquad (3)$$

For one particle the wave function $\psi(x)\to u+iv=w(z)$ is on the one-valued Riemann surface $f(z)=z$. This suggests that for two particles in the configuration space x_1, x_2 one has to set $f(z)=\sqrt{z}$, or

$$\left.\begin{aligned} x_1,x_2 &\to \pm\sqrt{x+iy}=\pm\sqrt{z} \\ \psi(x_1,x_2) &\to u+iv=w(\pm\sqrt{z}) \end{aligned}\right\} \qquad (4)$$

The generalization to N particles is obtained, setting

$$\left.\begin{aligned} x_1, x_2, \ldots, x_N &\rightarrow \sqrt[N]{x+iy} = \sqrt[N]{z} \\ \psi(x_1, x_2, \ldots, x_N) &\rightarrow u + iv = w(\sqrt[N]{z}) \end{aligned}\right\} \quad (5)$$

This means the motion in the N-dimensional configuration space is the wave function on one N-valued Riemann surface with the equation $f(z) = \sqrt[N]{z}$.

It is instructive to compare the motion along the x-axis for one particle with the motion of two particles. While the motion of a one-particle wave package along the x-axis can be illustrated as in Fig. 1.a, the description for two entangled particles moving along the x-axis in the opposite direction shown in Fig. 1.b is wrong. There the description would have to be in the abstract x_1, x_2 configuration space as shown in Fig.1c.

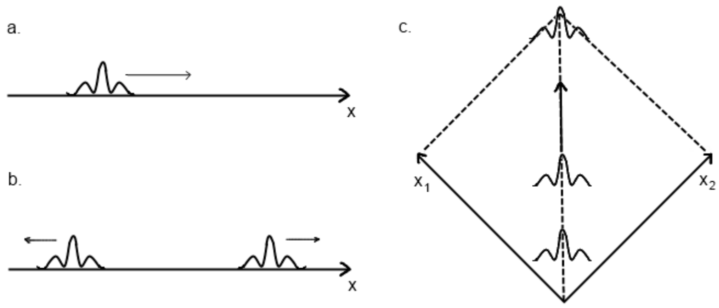

Figure 1: a. *One particle (correct).* **b.** *Two particles (incorrect).* **c.** *Two particles (correct: configuration space x_1, x_2).*

But on the Riemann surface $f(z) = \sqrt{z}$, shown in Fig. 2, it would be on the two surfaces, with the particles positioned on top of each other, as shown in Fig. 3. While the first particle moves here along the x-axis but on the upper branch $+\sqrt{z}$, the second particle also moves along the x-axis but on the lower branch $-\sqrt{z}$, just under the position of the first particle.

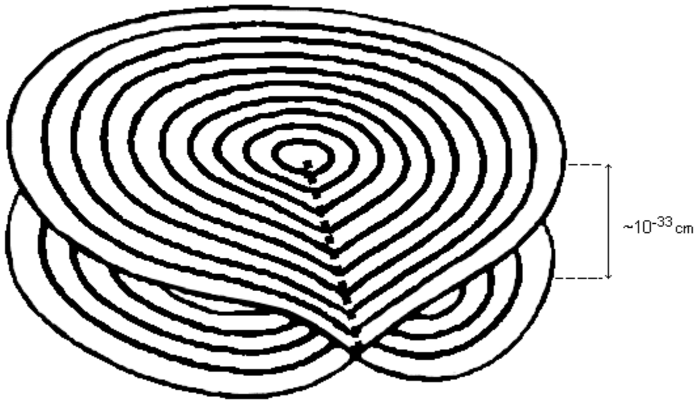

Figure 2: *The Riemann surface* $f(z)=\sqrt{z}$
(From K. Knopp, Funktionentheorie, Vol. II, p. 90, W. de Gruyter, Berlin 1949).

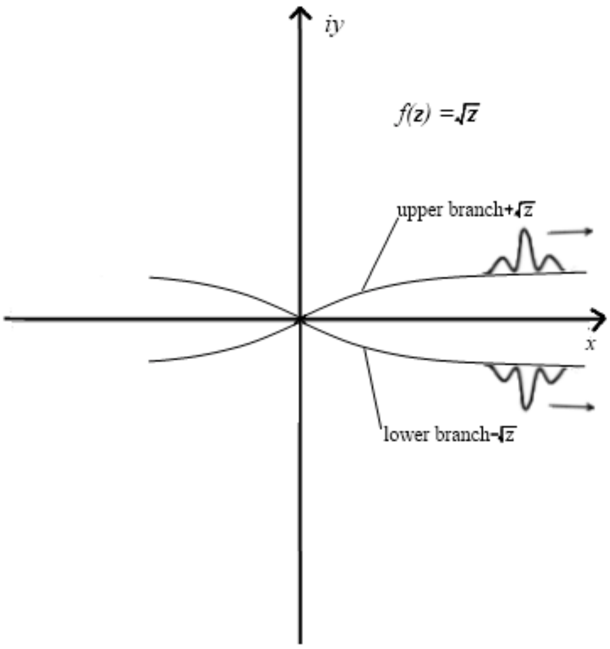

Figure 3: *Motion of two entangled particles in Riemann surface space.*

To obtain the distance of the first particle on the positive branch $+\sqrt{z}$ to the second particle on the negative branch $-\sqrt{z}$, one has to go on the positive branch in the negative direction from the position $x \rightarrow +\sqrt{z}$, $y = 0$, to the position $z = 0$, and from there on the negative branch $x \rightarrow -\sqrt{z}$ in the positive direction to the position of $x = -\sqrt{z}$, $y = 0$. The length $L = 2x$ is the distance of separation of the two entangled particles in the world above the Planck length. But below the Planck length there is a much shorter distance between the two particles by passing through a wormhole from the first particle on the upper branch to the second particle on the lower branch. If the conjecture that the configuration space is in reality a complex Riemann-Teichmüller space hidden in the space below the Planck length, the strange EPR correlations can become understandable, because the time to go from the upper plane of the Riemann surface, assumed to be separated by a Planck length $l \sim 10^{-33}$ cm from the lower surface, would be the Planck time (G Newton's constant) $t_p = \sqrt{\hbar G / c^5} \approx 10^{-44}$ s. It has been reported by Gisin that the quantum correlations in EPR experiments go faster than $10^4\, c$ [11]. Gisin's experiments were conducted over a distance of ~ 10 km. But if they go through a wormhole in the time of ~ 10^{-44} s, the apparent velocity over ~ 10 km = 10^6 cm would rather be $10^6/t_p \sim 10^{50}$ cm/s ~ $10^{40}\, c$, beyond what can be measured.

3 Generalizations

The extension to more than one space dimension would have to take into account the boundary condition and external forces on the wave-function, but the wave function would be hidden below the Planck length. And going from one to many particles would there have to be done by a mapping of the abstract configuration space into a complex Teichmüller space.

The Teichmüller space together with its metric is homeomorphic to a flat Euclidean space, as is the configuration space of quantum mechanics in the absence of gravitational forces. This is the consequence of the Teichmüller extremal mapping theorem [12]. In the presence of gravitational forces the Riemann surface and Teichmüller space are curved in accordance with the general theory of relativity.

4 Possible experimental verification

A possible verification is suggested by the following experiment: In performing an EPR experiment, a wormhole would be opened between the two entangled particles, keeping their quantum mechanical phases the same. Now suppose that intense laser light is projected on that part of the screen where one of the entangled particles is expected to emerge and be measured. Under these conditions a laser signal might be seen on the other screen where the other entangled particle is expected to emerge, with the laser signal having passed through the wormhole made by the entangled particles.

5 Conclusion

The proposed extension of quantum mechanics by the analytic continuation into a Teichmüller space permits retaining the deBroglie-Bohm causal interpretation of quantum mechanics by resolving the faster than light "spooky" action at a distance EPR paradox as a consequence of the multi-valued character of the wave function in the complex Teichmüller space below the Planck length.

Schrödinger said: "I would not call the entanglement one, but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought." If it should find its rational explanation in a conjectured Teichmüller space below the Planck length, then it would, with Mochizuki's discovery of a deep connection between the Teichmüller theory and number theory [3], give number theory an important place in the fundamental laws of nature.

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